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INTER SECTION GRAPH OF FINITE ABELIAN GROUP AND SUB GROUPS OF FINITE GROUPS

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ABSTRACT

In a graph theory we have use basis of the group like abelian group and subgroup for intersection. An intersection containing a non- unit element .We characterize certain classes of subgroup intersection graphs corresponding to finite abelian groups. We check all its solvable groups whose intersection graphs are triangle-free. Surrounded the other results, we analysis all abelian groups whose intersection graphs are complete. Finally, we study the intersection graphs of cyclic groups.

KEYWORDS:

Abelian group, Subgroup, Intersection graph, Trivial, vertex, Isolated, Frobenius, Kernal

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CHAPTER 1

DEFINITION 1.1.

Let G be a group. The intersection graph G (G) of G is the undirected graph (without loops and multiple edges) whose vertices are in a one-to-one correspondence with all proper non-trivial subgroups of G and two vertices are joined by an edge, if and only if the corresponding subgroups of G have a non- trivial intersection(ie, an intersection containing

a non- unit element).

We know that, for every finite cyclic group n, for each divisor d of n there exists a unique Sub group of order d.

Example.1.2.

Consider the cyclic group of order 12(i.e, Z_{12})

Soln.

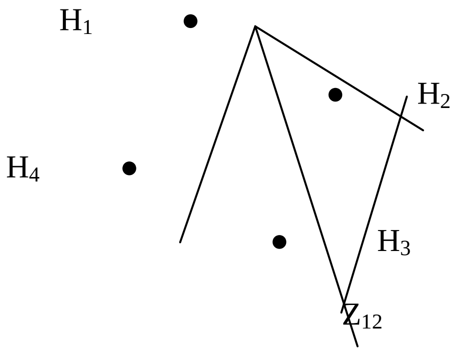
Z_{12} has 4 proper subgroups as follows:

H_1 = {0,2,4,6,8,10} ,

H_2 = {0,3,6,9} ,

H_3 = {0,4,8},

H_4 = {0,6}



Example 1.3.

Consider the cyclic group of order 36 (i.e, Z_{36})

Soln.

Z_{36} has proper subgroups as follows:

H_1 = {0,2,4 ……34},

H_2 = {0,3,6 ……33},

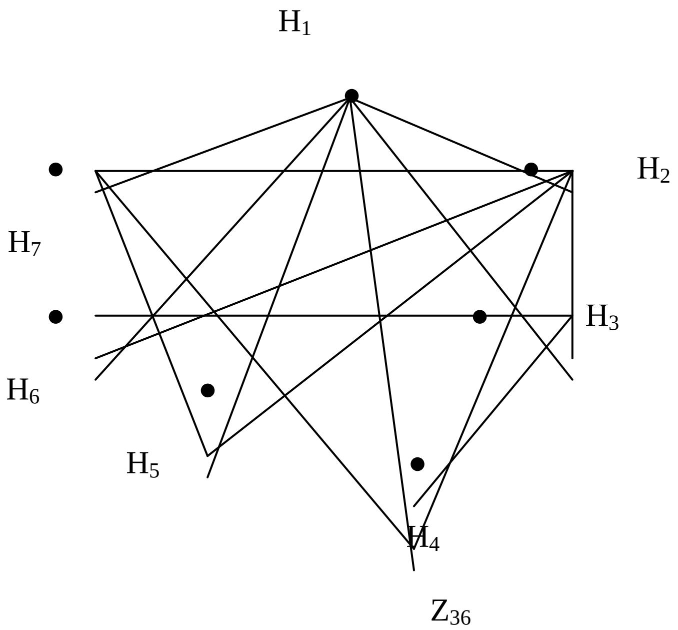
H_3 = {0,4,8 ……32} ,

$H_4 = \{0,6,12, \ldots..30\}$,

$H_5 = \{0,9,18,27\}$,

$H_6 = \{0,12,24\}$,

$H_7 = \{0,18\}$.



Example 1.4.

Consider cyclic group of order 30 (ie,Z_{30}).

Soln.

Z_{30} has 6 proper subgroups as follows:

$H_1 = \{0,2,4 \ldots \ldots 28\}$,

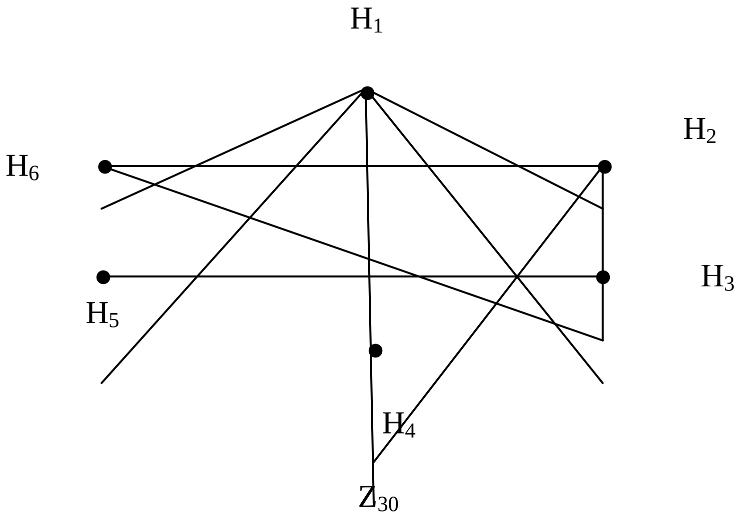
$H_2 = \{0,3,6 \ldots..27\}$,

$H_3 = \{0,5,10, \ldots 25\}$,

$H_4 = \{0,6,1218,24\}$,

$H_5 = \{0,10,20\}$,

$H_6 = \{0,15\}$.



Lemma 1.5.

Any finite non-trivial abelian group contains a cyclic subgroup whose order in a prime number.

Proof.

Any finite abelian group can be expressed as a direct product of a primary cyclic groups.

ie, cyclic groups of the order equal to a power of a prime number.

If a is the generator and p^{α} the order f any of these primary cyclic groups, then it is subgroup generated by $\alpha^{p\alpha-1}$ is cyclic and has the order p, which is a prime number.

Evidently, a primary cyclic group can contain only one such subgroup.

Lemma 1.6 .

The vertex independence number of the graph GG is equal to the maximal number of prime order subgroups of G.

Proof.

Two distinct prime order subgroups of G have always a trivial intersection .

Because such groups contain only one proper subgroup , namely the trivial one.

Therefore any system of prime order subgroups of G corresponds to an independent set in G (G). Now, let us have a maximal independent set in G (G). Any vertex of this set corresponds to a subgroup G : this subgroup has a prime order subgroup lemma, As any two subgroups of G corresponding to vertices of this independent set have trivial intersection , the prime order subgroups in subgroups of G corresponding to distinct vertices of this set must be distinct.

This implies that an independent set in G(G) corresponds to a subgroup of G containing more than one prime order subgroup, the cardinality of this independent set is less than the independence number of the graph G(G).

Corollary 1.7.

A vertex of G (G) corresponds to a primary cyclic subgroup of G. If and only if it belongs to some independent set of G(G) of maximal cardinality.

Lemma 1.8.

Let G be a finite abelian group which is not a direct of a product of two prime order groups .Let u, v be two vertices of G(G) not joined by an edge and corresponding to primary cyclic subgroups U , B of G(G). Then the orders of U and B are powers of different prime numbers, if and only if there exists a vertex w in G(G) joined with both u and v and with no vertex which is not joined with u and v.

Proof .

Let the orders of U and B be powers of different prime numbers.

Let W be the subgroup of G generated by the prime order subgroups of U and B .

The subgroup W is a proper subgroup of, because G is not a direct product of two prime order groups.

The vertex w of GG corresponding to (W) is evidently joined with both u and v.

Now let some vertex x of G (G) be joined with w .

This means that x correspondence to a subgroup X of G such that $X \cap W \neq \{e\}$.

Let $e \neq a \epsilon X \cap W$; then $a = b^m c^n$,where b ,c are generators of U and B respectively.

If p,q are orders of b,c respectively, take $a^p = b^{mp} c^{np}$.

This is equal to c^{np} , because $b^{mp} = c$.

According to the assumption , p ,q are relatively prime.

Therefore $c^{np} = e$ implies $np \equiv 0(mod\ (q))$ and $n \equiv 0\ (mod\ (q))$ which means $c^n = e$ and $a=b^m$.

We have either $a=b^m$,or $a^p = c^{np} \neq e$.

As both a and a^p are in (X),this means either $X \cap U \neq \{e\}$, or $X \cap B \neq \{e\}$ and x is joined either with u or with v.

Now, let the orders of U and B be powers of the same prime number p: Let the order of U be p^α, the order of B be p^3.

Without loss of generality , $\alpha \leq \beta$.

Let b,c be the generators of U and B respectively.

Then $c^{p\beta-\alpha}$ has the same order p^α as b and the product of $bc^{p\beta-\alpha}$ has also this order.

The primary cyclic subgroup generator by $bc^{p\beta-\alpha}$ will be denoted by W : evidently, it has trivial intersection with U and B.

Let S be a subgroup of G which has non- trivial intersection with both U and B;

Thus $X \cap U \ni b^r$, $X \cap B \ni c^8$, where r,s are positive integers, $r \equiv 0\ (modp^\alpha)$, $s \equiv 0\ (modp^\beta)$. Then X contains also the product $(bc^{p\beta-\alpha})$, where t is the least common multiple of r and of the greatest common divisor of $p^{\beta-\alpha}$ and s.

This element evidently different from e and belongs to W .

Therefore $X \cap W \neq \{e\}$ and x is joined also with w.

As X was chosen arbitrarily, the assertion is proved.

Hence the proof.

Lemma 1.9.

Let G be a direct product of two prime order groups. If these groups have different orders , the graph G (G) consists of two isolated vertices. If these groups have equal order, the graph G (G) contains more than two vertices.

Lemma 1.10.

Let G be a finite Abelian group whose order is a power of a prime number p.

Then the vertex independence number of G (G) is equal to $\sum_{i=0}^{n-1} p^i$, where n is the number of

direct factors in the expression of G as a direct product of primary cyclic groups.

Proof.

Let G_1.....G_n be the factors in the mentioned direct product.

Evidently G contains exactly one prime order subgroup S for i = 1,….n;

Therefore it contains p – 1 elements of prime order .

All elements of the order p are products of these elements;thus their number is $p^n - 1$.

As any prime order subgroup G has the order p and thus p – 1 non – unit elements which are all of the order p and as any two of such subgroup have trivial intersection, there are

$(p^n - 1)/(p^n - 1) = \sum_{i=0}^{n-1} p^i$ prime order subgroups of G.

According to lemma, this is also the vertex independence number of the graph G (G),

We can find $\sum_{i=0}^{n-1} p^i$ for any of these Sylow subgroups.

Theorem 1.11.

Let G be a finite Abelian group. Let G (G) be its intersection graph.

Knowing the graph G (G) , we can determine the number of factors in the expression of G as a direct product of Sylow groups and the intersection graph for any of these sylow groups.

Moreover, for any of these sylow subgroups of G, we can determine the number $\sum_{i=0}^{n-1} p^i$, where p is the prime number whose power is the order of this group and n the number of factors in its expression as a direct product of primary cyclic groups.

Proof .

Let G (G) be given .We find an independent set A of vertices in G (G) maximal cardinality: it corresponds to a system of primary cyclic subgroups of G with pairwise trivial intersection (Lemma and its corollary).

According to Lemma, we shall decide for an pair of vertices of A whether the orders of the subgroup of G corresponding to these vertices are powers of the same prime number or not .

Now, let B be a subset of A such that all vertices of B correspond to the subgroups ofG whose orders are powers of the same prime number p and any vertex of A – B corresponds to a subgroup whose order is a power of another prime number.

The subgraphs of G corresponding to vertices of A – B belong to other sylow subgroup.

The mentioned sylow subgroup contains as its non-trivial subgroups exactly all subgroups of G which have a non-trivial intersection with atleast one subgroup corresponding to a vertex of S and have trivial intersections with all subgroups corresponding to vertices A – B .

This can be proved simply .

The subgroups corresponding to vertices of B contain as their subgroups all subgroups of G of the order p (any of them contains exactly one such subgroup);

Therefore any subgroup of G of the order equal to a power of p must have a non-trivial intersection with some of them.

Now, if a subgroup of G has a non-trivial intersection with a subgroup corresponding to a vertex of A – B, this intersection contains an element whose order is equal to a power of a prime number different from p and thus this subgroup is not a subgroup of the mentioned sylow subgroup.

The intersection graph of this sylow subgroup is therefore the subgraph of S (G) induced by the

vertex set consisting of B and all vertices set of G(G) which are joined with atleast one vertices

of B with no vertex of A – B.

In this way we can construct intersection graphs of all sylow subgroups of G and thus also recognize the number of these subgroups .

According to Lemma,

Hence the proof

CHAPTER 2

INTERSECTION GRAPH OF SUBGROUPS

OF FINITE GROUPS

Definition 2. 1.

If there exist non trivial subgroups $L_1 \ldots L_n$ of G such that

$H \sim L_1$, $L_1 \sim L_2 \ldots L_{n-1} \sim L_n$, $L_n \sim K$,then we say that H and K are connected by the chain

$H \sim L_1 \sim L_2 \sim \ldots \sim L_n \sim K$. Clearly, in this case ρ (H , K) $\leq$ n+1.

The Dihedral group of order 2n.

$D_{2n} = < r, s > \{ 1, r, r^2, r^3 ,\ldots, r^{n-1}, s, sr, sr^2, sr^3 ,\ldots, sr^{n-1} \}$,

where $r^n = 1$, $s^2 = 1$ and $r^is = sr^{n-i}$.

Note 2.2.

For each positive integer n, let d(n) denote the number of positive divisors of n.

And let σ denote the sum of the positive divisors of n. The number of subgroups of dihedral group $D_{2n}(n \geq 3)$ is $d(n) + \sigma(n)$.

Example 2.3.

Consider the dihedral group order 8.

$D_8 = \langle r, s \rangle = \{ 1, r, r^2, r^3, r^4, s, sr, sr^2, sr^3, sr^4 \}$, where $r^4 = 1, s^2 = 1$

The proper sub group of D_8 are

$H_1 = \{1, r^2\}$,

$H_2 = \{1, r^2, s, sr^2\}$,

$H_3 = \{1, r^2, sr, sr^3\}$,

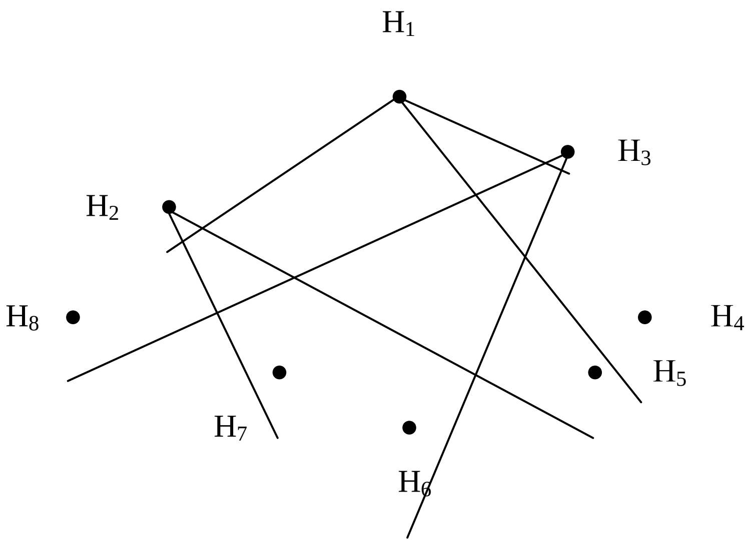
$H_4 = \{1, r, r^2, r^3\}$,

$H_5 = \{1, s\}$,

$H_6 = \{1, sr\}$,

$H_7 = \{1, sr^2\}$,

$H_8 = \{1, sr^3\}$.



D_8

Example 2.4.

The quarternion group of order 8.

Soln.

$Q_8 = \{\pm 1, \pm i, \pm j, \pm k\}$, where $ij = k$, $ji = -k$, $ik = j$, $ki = -j$, $jk = i$, $kj = -i$

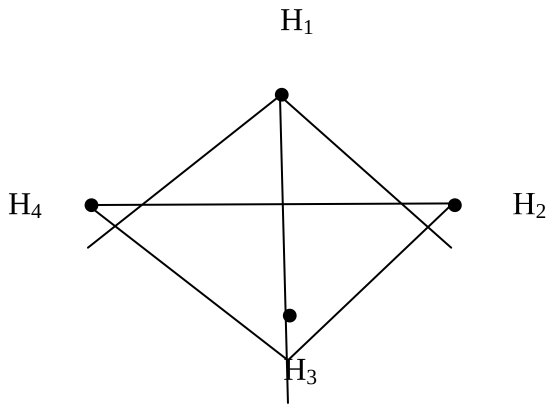
and $o(i) = o(j) = o(k) = 4$.

$H_1 = \{\pm 1\}$,

$H_2 = \{\pm 1, \pm i\}$,

$H_3 = \{\pm 1, \pm j\}$,

$H_4 = \{\pm 1, \pm k\}$.



Q_8

Example 2.5.

Consider the dihedral group order 12.

Soln.

$D_{12} = \langle r, s \rangle = \{1, r, r^2, r^3, r^4, r^5, r^6, s, sr, sr^2, sr^3, sr^4, sr^5, sr^6\}$,where $r^3 = 1$, $s^2 = 1$.

$H_1 = \{1, r^3\}$,

$H_2 = \{1, r^3, s, sr^3\}$,

$H_3 = \{1, r^2, r^4\}$,

$H_4 = \{1, s\}$,

$H_5 = \{1, sr\}$,

$H_6 = \{1, sr^2\}$,

$H_7 = \{1, sr^3\}$,

$H_8 = \{1, sr^4\}$,

$H_9 = \{1, sr^5\}$,

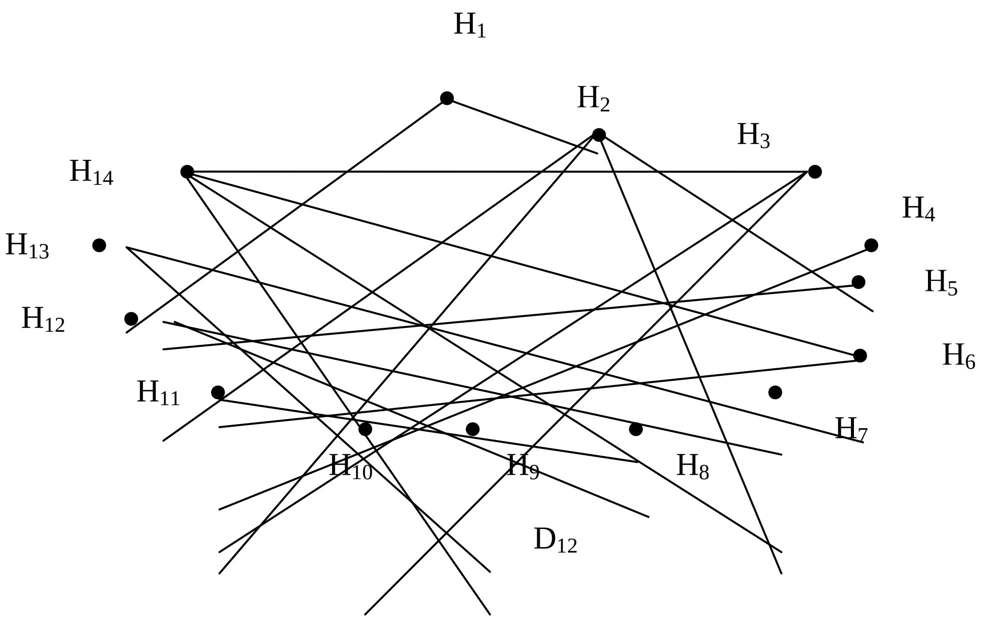
$H_{10} = \{1, r, r^2, r^3, r^4, r^5\}$,

$H_{11} = \{1, r^2, r^4, s, sr^2, sr^4\}$,

$H_{12} = \{1, sr, sr^3, sr^4\}$,

$H_{13} = \{1, r^3, sr^2, sr^5\}$,

$H_{14} = \{1, r^2, r^4, sr, sr^3, sr^5\}$



Lemma 2.6.

If G is connected, then the diameter $\delta(G)$ is equal to max$\{\rho$ (P,Q): both P,Q are subgroups of prime order of G$\}$.

Lemma 2.7.

Let B be a block of G and M be a proper subgroup of the group G .

If $B \cap M \neq 1$, then $M \subseteq B$.

Lemma 2.8.

Let B be a block of G .Then B is a subgroup of G or a normal subset of G.

Lemma 2.9.

Let G be disconnected and B = {B_1,B_2,…..B_l} be the set of all the subgroup blocks of G. Then any conjugate of B, is connected in B for i = 1….l.

Lemma 2.10.

If G is not a simple group, then one of the following cases occurs:

(1) The diameter δ (G) ≤ 4.

(2) G is $Z_p \times Z_q$, where p , q are primes.

(3) G is a Frobenius group whose complement is a group of prime order and the kernel is a minimal normal subgroup.

Proof.

Suppose that N is a non-trivial proper normal subgroup of G.

By Lemma, the required result δ (G) ≤ 4 is equivalent to (P,Q) ≤ 4 for any prime order subgroups P,Q with P ≠ Q.

Let $|P|$ = $|\langle a\rangle|$ = p and $|Q|$ = $|\langle b\rangle|$ = q.

Case 1:

PN = G.

(a) If Q ∩ N = $\langle b\rangle$ ∩ N = 1,then b ϵ G/N ≅ P , the order of every element of G\N is a multiple of p .

So the order of b is p ,that is o(b) = p = q.

If C_G (a) = G , then G = P × N ,Since o(a) = o(b) = p, we can assume that Q = $\langle (a, x)\rangle$, where x ϵ N and o(x) = p .

Now, we set H = {(y, z): y ϵ $\langle a\rangle$, z ϵ $\langle x\rangle$}.

If $|N|$ ≠ p , then H is a proper subgroup of G.

So that we have a chain P ~ H ~ Q.

Thus $\rho(P,Q) \leq 2$.

Certainly, when G is $Z_P \times Z_P$, there are p + 1 nontrivial.

(i.e., the intersection graph Γ (G) is the p + 1 isolated vertices graph .

If C_G (b) = G, then $\langle b \rangle \lhd G$.

Since b ≠ N, we have G = $\langle b \rangle \times N$ by virtue of $|G| = p|N|$.

So we can assume that $\langle a \rangle = \langle b, x \rangle$, where x ϵ N and o(x) = p.

Similarly we choose a group H = {(y, z): y ϵ $\langle b \rangle$, z ϵ $\langle x \rangle$}.

When $|N| \neq p$, then H is a proper subgroup of G .

So, P and Q are connected by a chain P – H – Q .

Thus we have also ρ (P,Q) ≤ 2.

Now we suppose that C_G (a) ≠ G and C_G (b) ≠G.

If $C_G(a) \cap N \neq 1$ and $C_G(b) \cap N \neq 1$,then $\langle a \rangle$ ~ $C_G(a)$ ~ N and $\langle b \rangle$ $C_G(b)$ ~ N,

So $\langle a \rangle$ ~ C_G (a) ~ N ~ C_G (b) ~ $\langle b \rangle$.

Then we have (P, Q) ≤ 4.

If C_G (a)∩ N = 1 or C_G (b) ∩ N = 1, we may assume without loss of generality,

That C_G (a) ∩ N = 1, then $\langle a \rangle$ acts non-fixed point on the subgroup N .

Thus G = N: $\langle a \rangle$ is a Frobenius group.

Clearly, if N is not a minimal normal subgroup of G, then we can choose a non-trivial

normal subgroup N_1 of N such that N_1 ~ G.

So we get a chain $\langle a \rangle$ ~ N_1 (a) ~ N_1 (b) ~ $\langle b \rangle$, hence we have ρ(P,Q) ≤ 3.

Certainly, if N is minimal normal subgroup of G, then G satisfies the requirement (3).

(b) Case $Q \leq N$

If C_G (a) = G (or C_G (b) = G), then p ⊳ G (or Q ⊳ G).

Hence, when PQ ≠ G, we have a chai n P ~ PQ ~ Q and then ρ (P, Q) ≤ 2.

Certainly, if PQ = G, then G = P × Q or G = Q: P is Frobenius group and

Hence the intersection graph of G is the empty graph on two or q+1 vertices.

Next, we consider the case of $C_G(a) \neq G$ and $C_G(b) \neq G$.

If $C_G(a) \cap N \neq \{1\}$, then $P \sim C_G(a) \sim N \sim Q$.

Hence, we have $\rho(P, Q) \leq 3$.

If $C_G(a) \cap N = \{1\}$, then P acts as a group N of fixed point free automorphism.

Thus G = N : <a> is a Frobenius group.

Similarity to the case (a), we have that N is a minimal normal subgroup of G.

Hence G satisfies the requirement (3).

Similarity, if QN = G, then we have the same results.

Case 2 :

$PN \neq G$ and $QN \neq G$.

P and Q can be joined by the same $P \sim PN \sim QN \sim Q$.

Thus $\rho(P, Q) \leq 3$.

Assertion I.

If n > 4, then the alternating group A_n is connected and $\delta(A_n) \leq 4$.

Proof.

By lemma, it suffices to prove that $\rho(P, Q) \leq 4$ for any subgroups P and Q of prime order.

Now, we can assume that P and Q are contained in maximal subgroups M_1 and M_2 respectively.

If $M_1 \cap M_2 \neq 1$, then $P \sim M_1 \sim M_2 \sim Q$, so that $\rho(P, Q) \leq 3$.

Next we will prove that the order of every maximal subgroup of A_n with $n \geq 5$ is more than n.

For the cases of n = 5 and 6, this is true by inspection

Now, suppose that $n \geq 7$.

Consider A_n in its natural degree n action.

If a maximal subgroup M is intransitive, say has an orbit of length k, then

$|M| \geq k!\,(n-k)!\,/2 > n$.

So M is transitive.

If $|M| = n$, then M is regular.

Each automorphism of M is induced by conjugation with some element from S_n.

This if M is maximal in A_n, then the automorphism group of M has order atmost 2.

Consider inner automorphisms , so the order of $M / Z(M)$ is less than or equal to 2,

hence M is abelian.

From $|Aut(M)| \leq 2$, we get $M = Z_n$ with $n = 2,3$ or 6,which is impossible.

Now return to our question.

If $M_1 \cap M_2 = 1$, we choose a largest maximal subgroup M of A_n, then it follows that

$M \cap M_1 \neq 1$ and $M \cap M_2 \neq 1$.

Indeed , otherwise ,if $M \cap M_1 = 1$, then $|MM_1| = |M||M_1| / |M \cap M_1| = |M||M_1| > n.A_{n-1} = |A_n|$,

a contradiction.

Hence $P \sim M_1 \sim M_2 \sim Q$, and consequently $\rho(P,Q) \leq 4$.

Assertion II.

If G is a simple group of lie type or a sporadic simple group, then its intersection graph is connected.

Proof.

Suppose that G has a disconnected intersection graphs.

Let the order of G be $p_1^{e1} p_2^{e2} \ldots . p_n^{en}$ and let $B_1, B_2 \ldots . B_k$ be blocks of G.

Now we choose a series of numbers $b_1, b_2 \ldots . b_k$ such that p_l^{el} llb_i if and only if there is an element of order p_l in B for l = 1,2….n and i = 1,2,….k .

By Lemma , if some B_i is a subgroup, then B_i is a maximal subgroup and B_i^g is also a block of

G for every g ϵ G.

On the other hand, $N_G(B_i) = B_i$ since B_i is maximal and B_i is not a maximal subgroup.

If follows that $N_G(B_i^g) = N_G(B_i)^g = B_i^g$,and hence $B_i \cap B_i^g = 1$ for all g ϵ G\$B_i$.

Thus G has a non-trivial normal subgroup by the well known Frobenius theorem ,

which contradicts the fact that G is a simple group .

So every B_i is a normal subset of G be Lemma.

Next we will prove, $(b_i, b_j) = 1$ for $i \neq j$.

If for some $1 \leq l \leq n$ and $1 \leq i , j \leq k$ there exists p_l such that p_l divides $(b_i, b_j) = 1$,

then there are a ϵ B_i , b ϵ B_j satisfying $o(a) = o(b) = p_l$.

Obviously, there exit sylow p_l subgroups P_1,P_2 of G containing a and b respectively.

Since P_1 and P_2 are conjugate ,we set $P_1^h = P_2$, then P_2 is contained in B_i by Lemma,

and hence B_i ,B_j are connected , a contradiction.

Therefore, $|G| = b_1, b_2 \ldots. b_k$ and a ϵ B_i if and only if o(a) divides b_i for any a ϵ G.

Choose M_i to be a maximal subgroup of G in the block B_i for i = 1, 2, … k .

By the above arguments we have $(|M_1|, |M_2|) = 1$ for $i \neq j$.

Hence for every prime pairs p_i , p_j ,where p_i divides b_i and p_j divides b_j for $i \neq j$,

we have that G has no element of order p_i , p_j.

Now we define another graph A (G) of G called the prime graph G, whose vertices set is

$\pi(G) = \{p:p$ is a divisor of $|G|\}$,vertices p and q in π (G) are joined by an edge if and only

if there exists an element of order pq.

The classification of disconnected prime graphs of non – abelian simple groups.

Now let $\pi(b_i)$ = {p:p is a prime divisor of b_i},then $\pi(b_i)$ is a prime graph component of G for i = 1, 2, … k .

Assume that 2 is contained in $\pi(b_1)$.

If G is a simple group of Lie type except $A_1(q)$, then M_i is a maximal torus of G for $i \geq 2$.

And hence $N_G(M_i) \cap B_1 \neq 1$,hence M_i is connected to M_1, a contradiction.

If G is $A_1(q)$ with q odd, set $\pi(b_2) = \pi(q) = p$,then M_2 is a elementary abelian p-group And M_2 is a sylow p subgroup of G,and we have $N_G(M_2) \neq M_2$ by the well-known Burnside theorem which states that a finite group G satisfying $N_G(P) = C_G(P)$ for some abelian sylow p group P is p-nilpotent.

Thus M_2 is not a maximal subgroup of G, a contradiction.

For the remaining cases when M_i of $A_1(q)$ for $i \geq 2$ is a maximal torus, we will get similar results.

If G is a sporadic simple group or $F_4(2)'$, the prime graph components vertices $\pi(b_i)$ with $i \geq 2$ form a single point set {p} and M_i is a cycle Sylow –p subgroup of G.

Clearly, M_i is not a maximal subgroup by the well-known Thomson theorem which asserts that a finite group having an odd order nilpotent maximal subgroup must be solvable.

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